1) Find the equation of the tangent line at the given point. $3y + 2y^2 + xy = 20$ (3,

$$3\frac{dy}{dx} + 4y\frac{dy}{dx} + y + x\frac{dy}{dx} = 0$$

$$3\frac{dy}{dx} + 4y\frac{dy}{dx} + x\frac{dy}{dx} = -y$$

$$\frac{dy}{dx}(3+4y+x) = -y$$

Slope

$$\frac{dy}{dx} = \frac{-y}{3+4y+x}$$

$$\frac{dy}{dx} = \frac{-2}{3+4(2)+3} = \frac{-2}{14}$$

$$m = \frac{-1}{7}$$

$$y = mx+b$$

$$2 = \frac{-1}{7}(3)+b$$

$$2 = \frac{-3}{7}+b$$

$$2+\frac{3}{7} = b$$

$$b = \frac{17}{7}$$

$$y = \frac{-1}{7}x + \frac{17}{7}$$

2) Find the derivative of $5xy^3 - 2x^3 = y$.

$$5y^3 + 5x(3y^2)\frac{dy}{dx} - 6x^2 = \frac{dy}{dx}$$

$$5y^3 - 6x^2 = 1\frac{dy}{dx} - 15xy^2\frac{dy}{dx}$$

$$5y^3 - 6x^2 = \frac{dy}{dx}(1 - 15xy^2)$$

$$\frac{5y^3 - 6x^2}{1 - 15xy^2} = \frac{dy}{dx}$$

3) Find the derivative of $\tan(xy^2) + 4x + 6y = 22$.

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$\tan(xy^2) + 4x + 6y = 22.$

$$\sec^2(xy^2)\left(y^2 + x(2y)\frac{dy}{dx}\right) + 4 + 6\frac{dy}{dx} = 0$$

$$y^2\sec^2(xy^2) + 2xy\cdot\sec^2(xy^2)\frac{dy}{dx} + 4 + 6\frac{dy}{dx} = 0$$

$$2xy\cdot\sec^2(xy^2)\frac{dy}{dx} + 6\frac{dy}{dx} = -y^2\sec^2(xy^2) - 4$$

$$\frac{dy}{dx}\left(2xy\cdot\sec^2(xy^2) + 6\right) = -y^2\sec^2(xy^2) - 4$$

$$\frac{dy}{dx} = \frac{-y^2\cdot\sec^2(xy^2) - 4}{2xy\cdot\sec^2(xy^2) + 6}$$

4) An inverted conical tank is being emptied at a rate of 12.6 ft^3/sec. The radius of tank is 7ft and the height of the tank is 24 ft. How fast is the height of the water changing when the depth is 13ft? How fast is the radius changing at the same insta

$$\frac{dV}{dt} = -12.6 \text{ ft}^3/\text{sec}$$

$$24r = 7h$$
$$24r = 7(13)$$
$$r = 3.792$$

$$\frac{r}{h} = \frac{7}{24}$$
$$24r = 7h$$
$$r = \frac{7}{24}h$$
$$h = \frac{24}{7} \cdot r$$

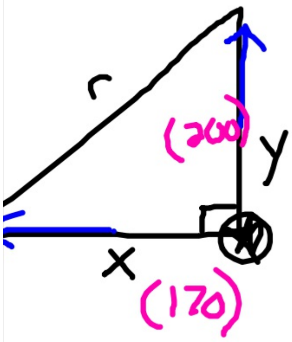
a) $V = \frac{\pi}{3} r^2 h$

$$V = \frac{\pi}{3}\left(\frac{7}{24}h\right)^2 \cdot h$$
$$V = \frac{49\pi}{1728} h^3$$
$$\frac{dV}{dt} = \frac{49\pi}{576} h^2 \frac{dh}{dt}$$
$$-12.6 = \frac{49\pi}{576}(13)^2 \frac{dh}{dt}$$
$$\frac{dh}{dt} = -.279 \text{ ft/sec}$$

b) $V = \frac{\pi}{3} r^2 h$

$$V = \frac{\pi}{3} \cdot r^2 \left(\frac{24}{7} r\right)$$
$$V = \frac{8\pi}{7} r^3$$
$$\frac{dV}{dt} = \frac{24\pi}{7} r^2 \frac{dr}{dt}$$
$$-12.6 = \frac{24\pi}{7}(3.792)^2 \frac{dr}{dt}$$
$$\frac{dr}{dt} = -.081 \text{ ft/sec}$$

5) A train is traveling away (north) from 30th Street Station in Philadelphia at a rate
85mph. Another train is traveling away (west) from 30th Street Station at a rate of
70mph. When the northbound train is 200 miles away and the westbound train is 17
from Philadelphia, how fast is the distance between the two trains changing?



$\frac{dy}{dt} = 85mph$

$\frac{dx}{dt} = 70mph$

$x^2 + y^2 = r^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2r\frac{dr}{dt}$

$2(170)(70) + 2(200)(85) = 2(262.488)\frac{d}{d}$

$\frac{dr}{dt} = 110.1mph$

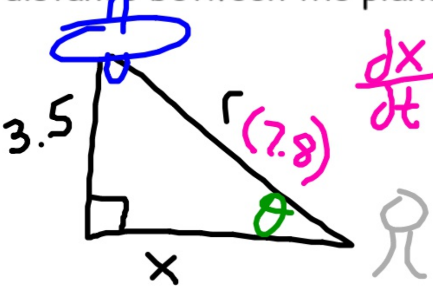
Helper

$x^2 + y^2 = r^2$

$170^2 + 200^2 = r^2$

$r = 262.488$

6) A plane flying horizontally at an altitude of 3.5 miles at a rate of 450 mph is flying towards a radar station. What is the rate of change of the angle of elevation when t distance between the plane and the station is 7.8 miles?



$$\frac{dx}{dt} = {}^{-}450\text{mph}$$

$$\tan\theta = 3.5x^{-1}$$

$$\sec^2\theta\frac{d\theta}{dt} = {}^{-}3.5x^{-2}\frac{dx}{dt}$$

$$\sec^2(.465)\frac{d\theta}{dt} = {}^{-}3.5(6.971)^{-2}(-450)$$

$$\frac{d\theta}{dt} = 25.894\text{ rad/hr.}$$

Helper

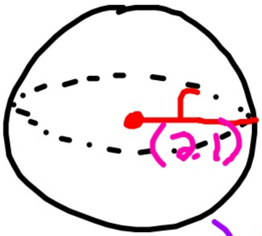
$$\sin\theta = \frac{3.5}{7.8}$$

$$\theta = .465$$

$$x^2 + 3.5^2 = 7.8^2$$

$$x = 6.971$$

7) A spherical balloon is being deflated at a rate of 2.75 cubic centimeters per minut
the instant the radius is 2.1 centimeters, what is the rate of change of the radius?
is the rate of change of the surface area at the same moment?



$$\frac{dV}{dt} = -2.75 \text{ cm}^3/\text{min}$$

a) $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$-2.75 = 4\pi(2.1)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = -.05 \text{ cm/min}$$

b) $SA = 4\pi r^2$

$$\frac{dSA}{dt} = 8\pi r \frac{dr}{dt}$$

$$\frac{dSA}{dt} = 8\pi(2.1)(-.05)$$

$$\frac{dSA}{dt} = -2.639 \text{ cm}^2/\text{min}$$